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Dynamic coupling of asymmetric shear wall structures: an analytical solution

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Abstract

In this paper, a dynamic analysis is presented for coupled flexural-warping torsional vibration of asymmetric shear wall structures in tall buildings. Due to the asymmetry of the structure, the free vibration is a coupled one, where laterally flexural vibrations in two orthogonal directions are coupled by a warping torsional vibration. Based on the continuum approach and D'Alembert's principle, the governing differential equation of free vibration and its corresponding eigenvalue problem for asymmetric shear wall structures are derived. Based on the theory of differential equations, an analytical method of solution is proposed to solve the eigenvalue problem and a general solution is derived for determining the natural frequencies and associated mode shapes of the structure. The proposed analysis is less approximate, and the numerical investigation pertaining to coupled vibration analysis of a generally asymmetric shear wall building shows that the results from the proposed analytical method and FEM analysis agree well. It is expected that the proposed analytical method of solution would enlarge the content of coupled vibration in the theory of dynamics of structures from theoretical research's point of view.

Keywords: Coupled vibration; Shear walls; Dynamic analysis; Tall buildings

1. Introduction

In a shear-wall tall building, the functional requirements generally result in asymmetric location of the structural elements, as shown in Fig. 1. In such structural configuration, the laterally flexural deflections in two orthogonal directions and the warping torsional rotation can no longer be treated separately due to their coupling in the governing differential equation of free vibration. In the past several decades, a large number of articles have been published on the coupled vibration analysis of asymmetric structures (Gere and Lin, 1958; Medearis, 1966; Heidebrecht and Raina, 1971; Rutenberg and Heidebrecht, 1975; Rutenberg and Tso, 1975; Kan and Chopra, 1977; Reinhorn et al., 1977; Rutenberg et al., 1977; Balendra et al., 1982, 1983, 1984; Zalka, 1995, 2000; Ng and Kuang, 2000). In most of these studies, the continuum approach was used to formulate the governing equations for the problem of torsional coupling including St. Venant and/or warping torsions of tall structures.

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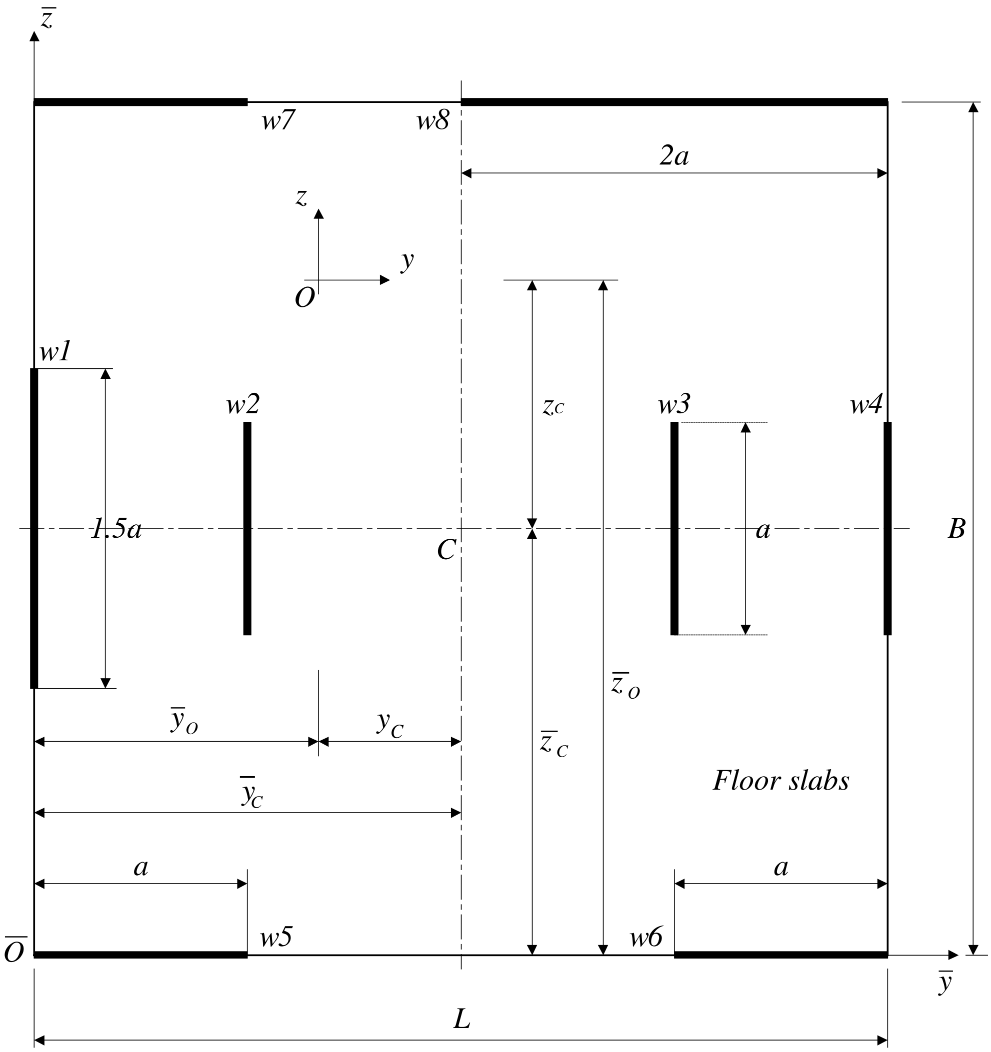

Fig. 1. Floor plan of a general asymmetric shear wall structure.

This paper presents a theoretical analysis of coupled flexural-warping torsional vibration for generally asymmetric shear-wall structures in tall buildings. The analysis includes the frequency and mode shape determinations for the coupled deflections (due to the laterally flexural deformation) and rotation (due to warping torsion deformation) of the structure. Based on the continuum approach and the D'Alembert's principle, the governing equation of the structure in free vibration and its corresponding eigenvalue problem, that is a set of the fourth-order ordinary differential equations for two laterally flexural vibrations coupled with a warping torsional vibration, are derived. Based on the theory of differential equations, an analytical method of solution is proposed to solve the eigenvalue problem and a general solution is derived for determining the natural frequencies and associated mode shapes of the structure. The proposed analytical method of solution is less approximate. A comparison is made between the results from the proposed method and FEM analysis of an example asymmetric shear-wall structure, and it is shown that two sets of results are in very good agreement.

The proposed analysis is derived from a uniform model of shear wall structures. In practice, the thickness of shear walls in a tall building varies over the entire height of the structure. However, the change rate of the walls is generally low along the structural height; an equivalent uniform structure may then generally be used in the design practice to replace the non-uniform one, particularly at the preliminary design stage, for the drift and dynamic analyses (Stafford Smith and Coull, 1991; Taranath, 1997).

2. Theory

A general asymmetric shear-wall structure of height H is shown in Fig. 1. In the analysis, the shear wall structure is considered as an equivalent flexural cantilever (Rutenberg et al., 1977; Balendra et al., 1984; Zalka, 1995), which is located at the centre of flexural rigidity, O (Kuang et al., 1991). Under the action of lateral loading, the flexural cantilever beam may undergo deformations of both lateral flexure and warping torsion (Stafford Smith and Crowe, 1985; Stafford Smith and Coull, 1991). The vertical x-axis is chosen over the structural height and through the point O, and the co-ordinate (y_C, z_C) represents the position of the geometric centre of the floor plan C in the yOz co-ordinate system. It is assumed that the structure has a uniformly distributed mass m, mass polar moment of inertia m_I, flexural stiffnesses EI_y and EI_z in y and z directions, respectively, and warping torsion stiffness EI_ω along the structural height. Details of calculation of flexural and torsional stiffnesses and geometric properties are given in Appendix A.

2.1. Governing equation for coupled vibration

Let $y(x,t)$ be the lateral deflection of the point O in y-direction, $z(x,t)$ the one in z-direction, and $\theta(x,t)$ the torsional rotation of the floor plan about the point O at the height x $(0 \leqslant x \leqslant H)$ and time t. Based on the D'Alembert's principle (Meirovitch, 1986), the governing equation of the natural vibration for a shear wall structure is derived conveniently by substituting inertial forces into the equations of static equilibrium:

$$\mathbf{E}\frac{\partial^4 \mathbf{X}(x,t)}{\partial x^4} + \mathbf{M}_0 \frac{\partial^2 \mathbf{X}(x,t)}{\partial t^2} = 0. \tag{1}$$

In Eq. (1), $\mathbf{X}$, $\mathbf{E}$ and $\mathbf{M}_0$ represent the displacement vector and flexural stiffness and mass matrices, respectively, given by

$$\mathbf{X} = \begin{Bmatrix} y(x,t) \\ z(x,t) \\ \theta(x,t) \end{Bmatrix}, \quad \mathbf{G} = \begin{bmatrix} EI_y & 0 & 0 \\ 0 & EI_z & 0 \\ 0 & 0 & EI_\omega \end{bmatrix}, \quad \mathbf{M}_0 = \begin{bmatrix} m & 0 & -mz_C \\ 0 & m & my_C \\ -mz_C & my_C & mr_m^2 \end{bmatrix}, \tag{2}$$

in which the inertial radius of gyration is

$$r_m = \sqrt{\frac{L^2 + B^2}{12} + y_C^2 + z_C^2}, \tag{3}$$

where L and B are the length and width of the floor plan as shown in Fig. 1.

As there are zero deflection and zero rotation at the fixed base, and zero moment and zero resultant shear at the free top of the structure, the corresponding boundary conditions of Eq. (1) are as follows:

$$\mathbf{X} = \frac{\partial \mathbf{X}}{\partial x} = 0 \ \text{ at } x = 0 \text{ and } \frac{\partial^2 \mathbf{X}}{\partial x^2} = \frac{\partial^3 \mathbf{X}}{\partial x^3} = 0 \ \text{ at } x = H. \tag{4}$$

The governing equation (1) is considered as a set of dynamic equilibrium equations, and can be written to the following three equations:

$$EI_y \frac{\partial^4 y(x,t)}{\partial x^4} + m \frac{\partial^2}{\partial t^2}[y(x,t) - z_C \theta(x,t)] = 0, \tag{5}$$

$$EI_z \frac{\partial^4 z(x,t)}{\partial x^4} + m \frac{\partial^2}{\partial t^2}[z(x,t) + y_C \theta(x,t)] = 0 \tag{6}$$

and

$$EI_\omega(x) \frac{\partial^4 \theta(x,t)}{\partial x^4} - m \frac{\partial^2}{\partial t^2}\left[z_C y(x,t) - y_C z(x,t) - r_m^2 \theta(x,t)\right] = 0. \tag{7}$$

Eqs. (5) and (6) describe that the sum of all forces applied on the floor plan in y and z directions should be equal to zero, i.e. $\sum F_y = 0$ and $\sum F_z = 0$; Eq. (7) describes that the sum of all moments about the centre of flexural rigidity O applied on the floor plan should be equal to zero, i.e. $\sum M_0 = 0$.

2.2. Eigenvalue problem

The motion in free vibration at any point of the structural height x is considered to be harmonic one, and the deflected shapes are independent of time t. The displacement vector can then be written, in a separable form of variables x and t, as

$$\mathbf{X}(x,t) = \mathbf{U}(u)\sin\omega t, \tag{8}$$

in which $u = x/H$, ω is the circular frequency, and the mode shape vector is

$$\mathbf{U}(u) = \left\{ \begin{array}{c} y(u) \\ z(u) \\ \theta(u) \end{array} \right\}. \tag{9}$$

Substituting Eq. (8) into Eq. (1) and carrying out the necessary differentiation lead to the eigenvalue equation of the asymmetric shear-wall structures in free vibration:

$$\mathbf{E}\mathbf{U}^{(4)}(u) - \omega^2 \mathbf{M}\mathbf{U}(u) = 0, \tag{10}$$

in which the mass matrix is

$$\mathbf{M} = H^2 \mathbf{M}_0. \tag{11}$$

The boundary conditions of Eq. (10) are

$$\mathbf{U}(u) = \mathbf{U}'(u) = 0 \text{ at } u = 0 \quad \text{and} \quad \mathbf{U}''(u) = \mathbf{E}\mathbf{U}'''(u) - \mathbf{G}\mathbf{U}'(u) = 0 \text{ at } u = 1. \tag{12}$$

3. Method of solution

3.1. Derivation

To obtain an analytical solution to the eigenvalue problem of the coupled vibration of asymmetric shear wall structures, Eq. (10) may be rewritten in the following form:

$$\mathbf{U}^{(4)}(u) - \omega^2 \mathbf{N}\mathbf{U}(u) = 0, \tag{13}$$

in which

$$\mathbf{N} = \begin{bmatrix} \beta_y^2 & 0 & -\beta_y^2 z_C \\ 0 & \beta_z^2 & \beta_z^2 y_C \\ -\beta_\theta^2 \frac{z_C}{r_m^2} & \beta_\theta^2 \frac{y_C}{r_m^2} & \beta_\theta^2 \end{bmatrix}, \tag{14}$$

where the structural parameters

$$\beta_y^2 = \frac{mH^4}{EI_y}, \quad \beta_z^2 = \frac{mH^4}{EI_z}, \quad \beta_\theta^2 = \frac{mH^4}{EI_\omega} r_m^2. \tag{15}$$

According to the theory of differential equations, the mode shape function given by Eq. (9) can be expressed as

$$\mathbf{U}(u) = \begin{Bmatrix} y(u) \\ z(u) \\ \theta(u) \end{Bmatrix} = \begin{Bmatrix} a \\ b \\ 1 \end{Bmatrix} C e^{pu}. \tag{16}$$

Substituting Eq. (16) into Eq. (13) yields

$$\begin{Bmatrix} a \\ b \\ 1 \end{Bmatrix} p^4 - \omega^2 \mathbf{N} \begin{Bmatrix} a \\ b \\ 1 \end{Bmatrix} = \begin{Bmatrix} a \\ b \\ 1 \end{Bmatrix} p^4 - \omega^2 \begin{Bmatrix} \beta_y^2 (a - z_C) \\ \beta_z^2 (b + y_C) \\ \beta_\theta^2 \left(-\frac{z_C}{r_m^2} a + \frac{y_C}{r_m^2} b + 1 \right) \end{Bmatrix} = 0. \tag{17}$$

Then,

$$a = -\frac{z_C}{\frac{1}{\beta_y^2}\left(\frac{p^2}{\omega}\right)^2 - 1}, \tag{18a}$$

$$b = \frac{y_C}{\frac{1}{\beta_z^2}\left(\frac{p^2}{\omega}\right)^2 - 1} \tag{18b}$$

and

$$\frac{1}{\beta_\theta^2}\left(\frac{p^2}{\omega}\right)^2 + \frac{z_C}{r_m^2} a - \frac{y_C}{r_m^2} b - 1 = 0. \tag{18c}$$

Substituting Eqs. (18a) and (18b) into Eq. (18c) yields

$$\frac{1}{\beta_\theta^2}\left(\frac{p^2}{\omega}\right)^2 - 1 - \frac{z_C^2}{r_m^2} \frac{1}{\frac{1}{\beta_y^2}\left(\frac{p^2}{\omega}\right)^2 - 1} - \frac{y_C^2}{r_m^2} \frac{1}{\frac{1}{\beta_z^2}\left(\frac{p^2}{\omega}\right)^2 - 1} = 0. \tag{19}$$

It can be seen that Eq. (19) is a characteristic equation of coupled vibrations with a third-order polynomial for $(p^2/\omega)^2$, and the nature of the roots p is such that the roots can be written in the following form:

$$\left(\frac{p^2}{\omega}\right)^2 = k_j^2 \quad (j = 1, 2, 3); \tag{20}$$

hence,

$$p^2 = \pm k_1\omega,\ \pm k_2\omega,\ \pm k_3\omega. \tag{21}$$

Letting

$$\lambda_j^2 = k_j\omega \quad (j = 1, 2, 3) \tag{22}$$

yields

$$\begin{aligned} &p_{1,2} = \pm\lambda_1, \quad p_{3,4} = \pm\mathrm{i}\lambda_1, \\ &p_{5,6} = \pm\lambda_2, \quad p_{7,8} = \pm\mathrm{i}\lambda_2, \\ &p_{9,10} = \pm\lambda_3, \quad p_{11,12} = \pm\mathrm{i}\lambda_3. \end{aligned} \tag{23}$$

This enables the mode shape vector $\mathbf{U}(u)$, expressed by Eq. (16), to be rewritten in the following form:

$$\mathbf{U}(u) = \begin{Bmatrix} y(u) \\ z(u) \\ \theta(u) \end{Bmatrix} = [\,\mathbf{K}_1(u) \quad \mathbf{K}_2(u) \quad \mathbf{K}_3(u)\,] \begin{Bmatrix} \mathbf{C}_1 \\ \mathbf{C}_2 \\ \mathbf{C}_3 \end{Bmatrix}, \tag{24}$$

where

$$\mathbf{C}_j = \begin{Bmatrix} c_{j1} \\ c_{j2} \\ c_{j3} \\ c_{j4} \end{Bmatrix} \tag{25a}$$

and

$$\mathbf{K}_j(u) = \begin{pmatrix} a_j\cosh\lambda_j u & a_j\sinh\lambda_j u & a_j\cos\lambda_j u & a_j\sin\lambda_j u \\ b_j\cosh\lambda_j u & b_j\sinh\lambda_j u & b_j\cos\lambda_j u & b_j\sin\lambda_j u \\ \cosh\lambda_j u & \sinh\lambda_j u & \cos\lambda_j u & \sin\lambda_j u \end{pmatrix}. \tag{25b}$$

From Eqs. (18a) and (18b), the coefficients in Eq. (25b) are

$$a_j = -\frac{z_C}{\frac{k_j^2}{\beta_y^2} - 1}, \qquad b_j = \frac{y_C}{\frac{k_j^2}{\beta_z^2} - 1}. \tag{26}$$

By substituting the mode shape vector $\mathbf{U}$ expressed by Eq. (24) and its different order derivatives into the boundary conditions Eq. (12), the following set of homogeneous linear algebraic equations are obtained:

$$\begin{Bmatrix} \mathbf{U}(0) \\ \mathbf{U}'(0) \\ \mathbf{U}''(1) \\ \mathbf{U}'''(1) \end{Bmatrix} = \begin{bmatrix} \mathbf{K}_1(0) & \mathbf{K}_2(0) & \mathbf{K}_3(0) \\ \mathbf{K}_1'(0) & \mathbf{K}_2'(0) & \mathbf{K}_3'(0) \\ \mathbf{K}_1''(1) & \mathbf{K}_2''(1) & \mathbf{K}_3''(1) \\ \mathbf{K}_1'''(1) & \mathbf{K}_1'''(1) & \mathbf{K}_1'''(1) \end{bmatrix} \begin{Bmatrix} \mathbf{C}_1 \\ \mathbf{C}_2 \\ \mathbf{C}_3 \end{Bmatrix} = \mathbf{0}, \tag{27}$$

in which the coefficient matrix is expressed by

$$\begin{Bmatrix} \mathbf{K}_j(0) \\ \mathbf{K}'_j(0) \\ \mathbf{K}''_j(1) \\ \mathbf{K}'''_j(1) \end{Bmatrix} = \begin{pmatrix} a_j & 0 & a_j & 0 \\ b_j & 0 & b_j & 0 \\ 1 & 0 & 1 & 0 \\ 0 & a_j\lambda_j & 0 & a_j\lambda_j \\ 0 & b_j\lambda_j & 0 & b_j\lambda_j \\ 0 & \lambda_j & 0 & \lambda_j \\ a_j\lambda_j^2\cosh\lambda_j & a_j\lambda_j^2\sinh\lambda_j & -a_j\lambda_j^2\cos\lambda_j & -a_j\lambda_j^2\sin\lambda_j \\ b_j\lambda_j^2\cosh\lambda_j & b_j\lambda_j^2\sinh\lambda_j & -b_j\lambda_j^2\cos\lambda_j & -b_j\lambda_j^2\sin\lambda_j \\ \lambda_j^2\cosh\lambda_j & \lambda_j^2\sinh\lambda_j & -\lambda_j^2\cos\lambda_j & -\lambda_j^2\sin\lambda_j \\ a_j\lambda_j^3\sinh\lambda_j & a_j\lambda_j^3\cosh\lambda_j & a_j\lambda_j^3\sin\lambda_j & -a_j\lambda_j^3\cos\lambda_j \\ b_j\lambda_j^3\sinh\lambda_j & b_j\lambda_j^3\cosh\lambda_j & b_j\lambda_j^3\sin\lambda_j & -b_j\lambda_j^3\cos\lambda_j \\ \lambda_j^3\sinh\lambda_j & \lambda_j^3\cosh\lambda_j & \lambda_j^3\sin\lambda_j & -\lambda_j^3\cos\lambda_j \end{pmatrix}. \tag{28}$$

The solution of Eq. (27) consists of two parts. The first part is the eigenvalue, which corresponds to the natural frequency of coupled vibration. The second part is the eigenvector, which corresponds to the vibration mode shape. An analytical method of solution, which is based on the theory of differential equations, has been developed to estimate the natural frequencies and associated mode shapes of the asymmetric shear wall structures in coupled vibration.

3.2. Natural frequencies and associated mode shapes

Mathematically, the nontrivial solution to Eq. (27) can only be obtained when the determinant of the coefficients vanishes, i.e.

$$\begin{vmatrix} \mathbf{K}_1(0) & \mathbf{K}_2(0) & \mathbf{K}_3(0) \\ \mathbf{K}'_1(0) & \mathbf{K}'_2(0) & \mathbf{K}'_3(0) \\ \mathbf{K}''_1(1) & \mathbf{K}''_2(1) & \mathbf{K}''_3(1) \\ \mathbf{K}'''_1(1) & \mathbf{K}'''_1(1) & \mathbf{K}'''_1(1) \end{vmatrix} = d\lambda_1^2\lambda_2^2\lambda_3^2(1+\cosh\lambda_1\cos\lambda_1)(1+\cosh\lambda_2\cos\lambda_2)(1+\cosh\lambda_3\cos\lambda_3) = 0. \tag{29}$$

For the case of triple coupling, the coefficient

$$d = 4(a_1b_2 + a_2b_3 + a_3b_1 - b_1a_2 - b_2a_3 - b_3a_1)^4 \neq 0; \tag{30}$$

for the case of double coupling, the coefficient

$$d = 4(b_1 - b_2)^4 \neq 0. \tag{31}$$

The values of λ_1, λ_2, λ_3 should not be equal to zero at the same time, then

$$1 + \cosh\lambda_j\cos\lambda_j = 0 \quad (j = 1, 2, 3). \tag{32}$$

The solutions of Eq. (32) are therefore obtained

$$\left.\begin{array}{l} \lambda_1^{(1)} = \lambda_2^{(1)} = \lambda_3^{(1)} = 1.875 \\ \lambda_1^{(2)} = \lambda_2^{(2)} = \lambda_3^{(2)} = 4.694 \\ \lambda_1^{(3)} = \lambda_2^{(3)} = \lambda_3^{(3)} = 7.855 \\ \lambda_1^{(4)} = \lambda_2^{(4)} = \lambda_3^{(4)} = 10.996 \\ \lambda_1^{(i)} = \lambda_2^{(i)} = \lambda_3^{(i)} \approx \left(i - \frac{1}{2}\right)\pi \quad (i = 5, 6, \ldots, n, \ldots) \end{array}\right\}. \tag{33}$$

Therefore, from Eq. (22) the coupled frequencies of a general asymmetric shear-wall structure can then be calculated by

$$\omega_j^{(i)} = \frac{\lambda_j^2}{k_j}, \tag{34}$$

where i $(= 1, 2, 3, \ldots, n, \ldots)$ is the vibration mode number and j $(= 1, 2, 3)$ is the vibration shape number.

In the vibration shape function (24), the unknown constants c_{jl} $(j = 1, 2, 3$ and $l = 1, 2, 3, 4)$ expressed in Eq. (25a) can be determined from Eq. (27).

4. Numerical example

In order to validate and illustrate the proposed method and the computation procedure, an numerical investigation is presented of the coupled vibration analysis for a general asymmetric shear-wall building of height $H = 75$ m, consisting of 25 stories of 3-m high, with the plan arrangement of $L = B = 24$ m, as shown in Fig. 1. The structure consists of eight walls of 0.25-m thick and 6-m long ($a = 6$ m). An elastic modulus $E = 20 \times 10^6$ kN/m^2 and the intensity of floor slabs $\rho = 2{,}350$ kg/m^3 are assumed for normal concrete properties. The thickness of floor slab is 0.15 m. The natural frequencies of coupled lateral-torsional vibration and associated mode shapes are determined as follows:

(1) Values of flexural and torsional moments of inertia I_y, I_z and I_ω for all the shear walls are given in Table 1. The stiffnesses EI_y, EI_z, EI_ω are then determined using Eqs. (A.1) and (A.2) in Appendix A,

$$EI_y = 990.70 \times 10^6 \text{ kN m}^2, \quad EI_z = 574.53 \times 10^6 \text{ kN m}^2, \quad EI_\omega = 136.66 \times 10^9 \text{ kN m}^4.$$

The location of the geometric centre C of the floor plan is determined using Eqs. (A.1) and (A.3),

$$y_C = 4.463 \text{ m} \quad \text{and} \quad z_C = -7.631 \text{ m}.$$

The characteristic structural parameters β_y, β_z, and β_θ are calculated by employing Eq. (15),

$$\beta_y = 1.470 \text{ s}, \quad \beta_z = 1.931 \text{ s}, \quad \beta_\theta = 1.652 \text{ s}.$$

(2) By substituting values of the geometric properties and characteristic parameters of the structure into Eq. (19), the characteristic equation is expressed:

$$0.366\left(\frac{p^2}{\omega}\right)^2 - 1 - \frac{0.114}{0.268\left(\frac{p^2}{\omega}\right)^2 - 1} - \frac{0.334}{0.463\left(\frac{p^2}{\omega}\right)^2 - 1} = 0.$$

Table 1
Flexural and tortional moments of inertia of shear walls

Wall i	$\bar{y}_i$	$\bar{z}_i$	$I_{y,i}$ (m^4)	$I_{z,i}$ (m^4)	$I_{\omega,i}$ (m^6)
1	0	12	11.72×10^{-3}	15.19	863.46
2	6	12	7.81×10^{-3}	4.50	11.09
3	18	12	7.81×10^{-3}	4.50	493.08
4	24	12	7.81×10^{-3}	4.50	1,220.07
5	3	0	4.50	7.81×10^{-3}	1,734.34
6	21	0	4.50	7.81×10^{-3}	1,735.60
7	3	24	4.50	7.81×10^{-3}	86.06
8	19.5	24	36.00	15.63×10^{-3}	689.43
		Σ_i	49.54	28.73	6,833.13

Table 2
Circular frequencies of coupled vibration for the first three modes

Mode i	$\lambda_j^{(i)}$	$\omega_1^{(i)}$	$\omega_2^{(i)}$	$\omega_3^{(i)}$
1	1.875	1.622	2.004	3.843
2	4.694	10.163	12.556	24.083
3	7.855	28.457	35.156	67.432

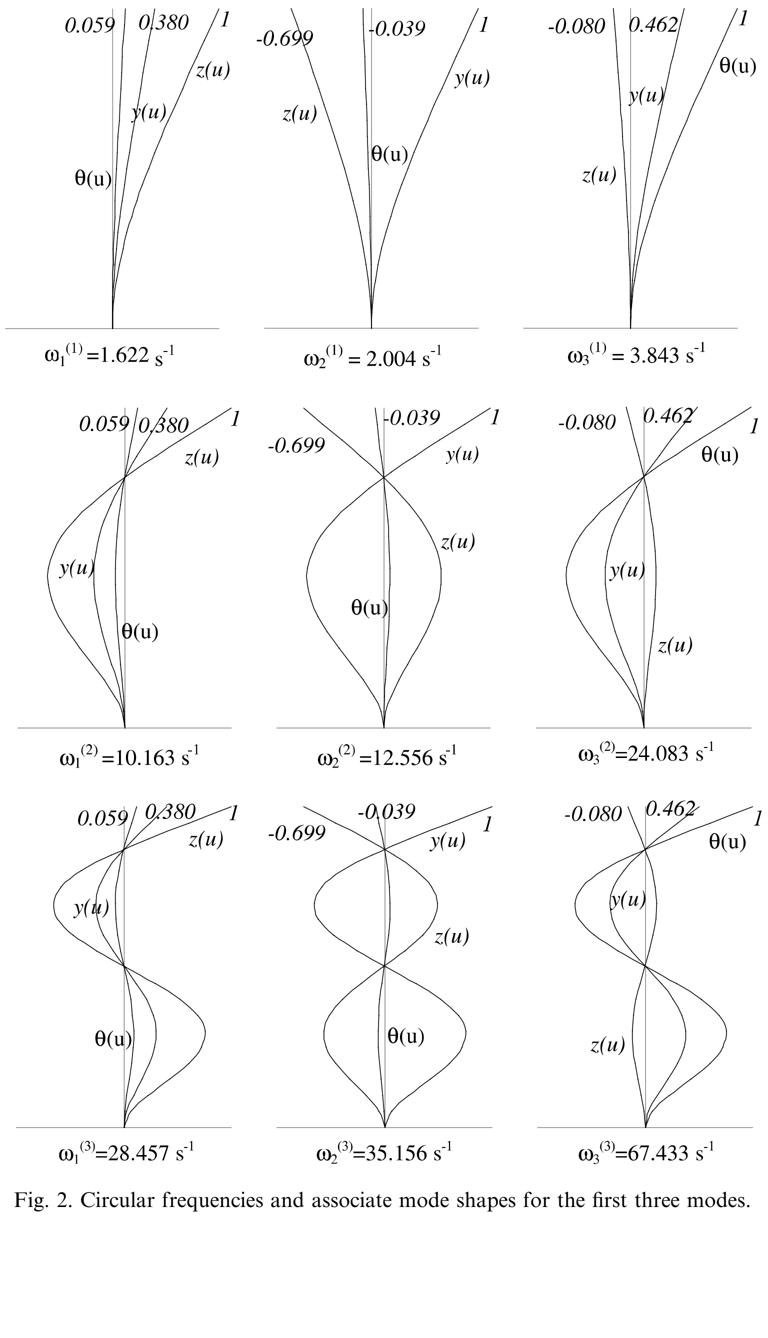

Fig. 2. Circular frequencies and associate mode shapes for the first three modes.

Table 3
Comparison of coupled frequencies between proposed method and FEM analysis

	Mode i	$\omega_1^{(i)}$	$\omega_2^{(i)}$	$\omega_3^{(i)}$
1	Proposed method	1.622	2.004	3.843
	FEM analysis	1.592	1.963	3.753
2	Proposed method	10.163	12.556	24.083
	FEM analysis	9.7943	12.207	22.906
3	Proposed method	28.457	35.156	67.432
	FEM analysis	27.292	33.395	63.724

The three roots of the characteristic equation are then obtained:

$$\left(\frac{p^2}{\omega}\right)^2 = \begin{cases} 4.701 \\ 3.080\,. \\ 0.837 \end{cases}$$

From Eq. (20), $k_1 = 2.168$, $k_2 = 1.755$, and $k_3 = 0.915$.

(3) The first three circular frequencies of the structure in coupled vibration are calculated using Eq. (34) and presented in Table 2. The associated mode shapes can be obtained using Eqs. (24)–(27) and are plotted in Fig. 2.

A comparison of coupled vibration frequencies of the structure is made in Table 3 between the results from the proposed analytical method of solution and a finite element analysis. The finite element analysis is performed by employing the computer program ETABS (Wilson et al., 1995), in which the multi-bent walls are considered as an assembly of discrete members comprised of finite column and beam elements. It can be seen from Table 3 that two sets of result from the proposed method and FEM analysis agree well.

5. Conclusions

Coupled flexural–warping torsional vibration analysis is presented in this paper for generally asymmetric shear-wall structures in tall buildings. Owing to the asymmetry of the structure the free vibration is a coupled vibration, where lateral flexure vibrations in two orthogonal directions are coupled by a warping torsion vibration about the vertical axis. Based on the continuum approach and D'Alembert's principle, the governing differential equation of free vibration and the corresponding eigenvalue equation to the problem are derived. Based on the theory of differential equations, an analytical method of solution is proposed and a general solution to the eigenvalue problem is derived for determining the coupled natural frequencies and associated mode shapes. The numerical investigation pertaining to coupled vibration analysis of an asymmetric, multi-bent shear-wall building is presented, and the results from the proposed analytical method are in good agreement with those from a comprehensive package programme for analysis of building structures. The proposed method is less approximate and provides an efficient means for dynamic analysis of asymmetric shear wall structures in coupled vibration. From the viewpoint of theoretical research, the proposed analytical method of solution would be considered to enlarge the content of coupled vibration in the theory of dynamics of structures.

Appendix A

A general asymmetric shear-wall structure shown in Fig. 1 can be represented by an equivalent cantilever beam, deforming in both lateral flexure and warping torsion. The cantilever is considered to locate at the centre of flexural rigidity O, whose location in an arbitrarily selected co-ordinate system $(\bar{y}, \bar{z})$ is given by

$$\bar{y}_{\rm O} = \frac{\sum_i \bar{y}_i EI_{z,i}}{\sum_i EI_{z,i}}, \qquad \bar{z}_{\rm O} = \frac{\sum_i \bar{z}_i EI_{y,i}}{\sum_i EI_{y,i}}, \tag{A.1}$$

where the co-ordinates $(\bar{y}_i, \bar{z}_i)$ represent the location of the centre O_i of the ith wall in the $(\bar{y}_i, \bar{z}_i)$ co-ordinate system.

The flexural stiffness and warping stiffness of the equivalent cantilever are

$$EI_y = \sum_i EI_{y,i}, \quad EI_z = \sum_i EI_{z,i}, \quad EI_\omega = \sum_i \left\lfloor \left(\bar{y}_j - \bar{y}_{\rm O}\right)^2 EI_{z,i} + \left(\bar{z}_j - \bar{z}_{\rm O}\right)^2 EI_{y,i} \right\rfloor, \tag{A.2}$$

where $EI_{y,i}$ and $EI_{z,i}$ are the flexural stiffnesses in y and z directions of the ith wall in its local co-ordinate system.

The vertical axis x is chosen over the structural height and through the point O, and the axes y and z are respectively parallel to $\bar{y}$ and $\bar{z}$ as the reference co-ordinate. The location of the geometric centre C of the uniform floor slabs in the co-ordinate system $y{\rm O}z$ is given by

$$y_{\rm C} = \bar{y}_{\rm C} - \bar{y}_{\rm O}, \qquad z_{\rm C} = \bar{z}_{\rm C} - \bar{z}_{\rm O}, \tag{A.3}$$

where the co-ordinate $(\bar{y}_{\rm C}, \bar{z}_{\rm C})$ is the location of the point C in the co-ordinate system $(\bar{y}, \bar{z})$.

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